



Soft Set and Soft Groups

By

Muhsin Tahsin

Email: **mtn377@maths.soran.edu.iq**

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Supervisor: Dr. Ismael

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Muhsin Tahsin

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Abstract

Molodtsov introduced the concept of soft set theory, which can be used as a generic mathematical tool for dealing with uncertainty. In this paper we introduce the basic properties of soft sets, and compare soft sets to the related concepts of fuzzy sets and rough sets. We then give a definition of soft groups, and derive their basic properties using Molodtsov's definition of the soft sets.

Introduction

Most of the problems in engineering, medical science, economics, environments etc. have various uncertainties. To exceed these uncertainties, some kinds of theories were given like theory of fuzzy sets , intuitionistic fuzzy sets , rough sets , i.e., which can be used as mathematical tools for dealing with uncertainties. As was mentioned , these theories have their own difficulties. In 1999, Molodsov initiated a novel concept of soft set theory, which is a completely new approach for modeling vagueness and uncertainty.

A soft set is a parameterized family of subsets of the universal set. We can say that soft sets are neighborhood systems, and that they are a special case of contextdependent fuzzy sets. In soft set theory the problem of setting the membership function, among other related problems, simply does not arise. This makes the theory very convenient and easy to apply in practice. Soft set theory has potential applications in many different fields, including the game theory, operations research, Fiemann integration, Perron integration, probability theory, and measurement theory. After Molodsov's work, some different applications of soft sets were studied M. Aslam et al./Ann. Fuzzy Math. Inform. 4 (2012), No. 1, 177–195 . Furthermore Maji, Biswas and Roy worked on soft set theory . Some corrections were given by M. Irfan et al. In this paper, we extend the idea of soft groups introduced by Aktas and Cagman and develop the soft group theory.

Chapter 1

Concept of Soft Set

Soft set is a parametrized general mathematical tool which deal with a collection of approximate descriptions of objects. Each approximate description has two parts, a predicate and an approximate value set. In classical mathematics, a mathematical model of an object is constructed and define the notion of exact solution of this model. Usually the mathematical model is too complicated and the exact solution is not easily obtained. So, the notion of approximate solution is introduced and the solution is calculated. In the soft set theory, we have the opposite approach to this problem. The initial description of the object has an approximate nature, and we do not need to introduce the notion of exact solution. The absence of any restrictions on the approximate description in soft set theory makes this theory very convenient and easily applicable in practice. Any parameterization we prefer can be used with the help of words and sentences, real numbers, functions, mappings and so on.

Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subset E$.

Definition 1.1. A pair (F, A) is called a soft set over U , where F is a mapping given by $F : A \rightarrow P(U)$.

In other words, a soft set over U is a parametrized family of subsets of the universe U

Definition 1.2. Let F be a soft set. A function $a : A \rightarrow U$ is called soft element of F if $a(t) \in F(t)$ for all $t \in A$. we write $a \in_s F$.

Definition 1.3. Let $F : A \rightarrow P(U)$ be a soft set. Then the collection of all soft elements of F is denoted by $SE(F)$, so $SE(F) = \{f : f : A \rightarrow U, f(t) \in F(t), \forall t \in A\}$

Theorem 1.4. For any two soft sets F and H , $H \subset_s F \Leftrightarrow SE(H) \subset SE(F)$.

Proof: Let $H \subset_s F$ then $H(t) \subset F(t)$ for all $t \in A$.

Now let $x \in SE(H)$, So $x : A \rightarrow U$ and $x(t) \in H(t)$ for all $t \in A$.

It implies that $x(t) \in F(t)$ as $H(t) \subset F(t)$.

Therefore $x \in SE(F)$.

Conversely, suppose $SE(H) \subset SE(F)$ and $t \in A$.

Let $\alpha \in H(t)$ and $\alpha \in_s H$ such that $a(t) = \alpha$. Then $a \in SE(H) \Rightarrow a \in SE(F)$.

Therefore $a(t) \in F(t)$.So $H(t) \subset F(t)$. Since $t \in A$ is arbitrary the result follows.

Example 1.5. Let (X, τ) be a topological space , that is X is a set and τ is a topology , in other words , τ is a family of subsets of X , called the open sets of X . Then the family of open neighborhoods $T(x)$ of point x , where $T(x) = \{V \in \tau \mid x \in V\}$, may be considered as the soft set $(T(x), \tau)$.

Example 1.6. Suppose that $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ be a universal set consisting of a set of six houses under consideration , $E = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters with respect to U , where each parameter $e_i, i = 1, 2, \dots, 5$ stands for expensive , beautiful ,cheap , modern , wooden respectively and $A = \{e_1, e_3, e_5\} \subset E$. Suppose a soft set (F, E) describes the attractions of the houses , such that

$$\begin{aligned} F(e_1) &= \{h_2, h_4\} \\ F(e_2) &= \{h_1, h_3\}, \\ F(e_3) &= \{h_3, h_4, h_5\}, \\ F(e_4) &= \{h_3, h_4, h_5\}, \\ F(e_5) &= \{h_1\} \end{aligned}$$

The soft set (F, E) is a parametrized family $\{F(e_i), i = 1, 2, 3, \dots, 5\}$ of subsets of the set U defined as

$$(F, A) = \{F(e_1), F(e_2), F(e_3)\} \quad \text{i.e.} \quad (F, A) = \{\{h_2, h_4\}, \{h_1, h_3, h_5\}, \{h_3, h_4, h_5\}\}$$

Example 1.7. Zadeh's fuzzy set may be considered as a special case of the soft set. Let A be a fuzzy set, and μ_A be the membership function of the fuzzy set A , that is μ_A is a mapping of U into $[0, 1]$.

Let us consider the family of α -level sets for function μ_A

$$F(\alpha) = \{x \in U \mid \mu_A(x) \geq \alpha\} \quad , \quad \alpha \in [0, 1]$$

If we know the family F , we can find the functions $\mu_A(x)$ by means of the following formulae:

$$\mu_A(x) = \sup_{\alpha \in [0, 1]} \alpha.$$

Thus, every Zadeh's fuzzy set A may be considered as the soft set $(F, [0, 1])$.

Definition 1.8. A soft set (F, A) over U is said to be empty whenever $A = \phi$.

Symbolically, we write (ϕ, ϕ) for the empty soft set over U .

Definition 1.9. A soft set (F, A) over U is called a universal soft set if $A = E$ and $F(e) = U$ for all $e \in A$. Symbolically, we write (U, E) for the universal soft set over U .

Definition 1.10. For two soft sets (F, A) and (G, B) over a common universe U , we say that (F, A) is a soft subset of (G, B) if

- (i) $A \subset B$, and
- (ii) $\forall e \in A$, $F(e)$ and $G(e)$ are identical approximations.

We write $(F, A) \subset^\sim (G, B)$.

(F, A) is said to be a soft super set of (G, B) , if (G, B) is a soft subset of (F, A) . We denote it by $(F, A) \supset^\sim (G, B)$.

Definition 1.11. Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A) .

Proposition 1.12. For any soft set (F, A) over U , $(\phi, \phi) \subseteq (F, A) \subseteq (U, E)$

Example 1.13. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be a universe set and

$E = \{e_1, e_2, e_3, e_4, e_5\}$ be a set of parameters.

Let $A = \{e_1, e_3, e_5\} \subset E$ and $B = \{e_1, e_2, e_3, e_5\} \subset E$

Let (F, A) and (G, B) be two soft sets over the same universe U where

$$G(e_1) = \{u_2, u_4\}, \quad G(e_2) = \{u_1, u_3\}, \quad G(e_3) = \{u_3, u_4, u_5\}, \quad G(e_5) = \{u_1\}$$

$$F(e_1) = \{u_2, u_4\}, \quad F(e_3) = \{u_3, u_4, u_5\}, \quad F(e_5) = \{u_1\}$$

Then $(F, A) \subset^\sim (G, B)$ since $A \subset B$ and $F(e) \subset G(e)$, $\forall e \in A$. But $(G, B) \not\subset^\sim (F, A)$

Hence $(F, A) \neq (G, B)$

1.1 Operations on Soft Sets

In this section, we generalize the basic operations on classical sets to soft sets.

soft set (F, A) is called a null soft set if $F(e) = \phi$ for all $e \in A$. Because ϕ does not belong to the codomain of F in our framework, we redefine the concept of empty soft set as follows.

Definition 1.14. Let (F, A) and (G, B) be two soft sets over U . The Intersection of (F, A) and (G, B) , denoted by $(F, A) \cap (G, B)$, is defined as $(F \cap G, C)$ where

$$C = \{e \in A \cap B \mid F(e) \cap G(e) \neq \phi\} \quad , \quad \forall e \in C$$

$$(F \cap G)(e) = F(e) \cap G(e)$$

In particular, if $A \cap B = \phi$ or $F(e) \cap G(e) = \phi$ for every $e \in A \cap B$, then we see that $(F, A) \cap (G, B) = (\phi, \phi)$

Definition 1.15. Let (F, A) and (G, B) be two soft sets over U . The Union of (F, A) and (G, B) , denoted by $(F, A) \cup (G, B)$, is defined as $(F \cup G, C)$ where

$$C = A \cup B \quad , \quad \forall e \in C$$

$$(F \cup G)(e) = \begin{cases} F(e) & \text{if } e \in A \setminus B \\ G(e) & \text{if } e \in B \setminus A \\ F(e) \cup G(e) & \text{Otherwise} \end{cases}$$

Definition 1.16. Let (F, A) be a soft sets over U . The Complement of (F, A) ,with respect to the universal soft set (U, E) , denoted by $(F, A)^c$, is defined as (F^c, C) where

$$C = E \setminus \{e \in A \mid F(e) = U\} = \{e \in A \mid F(e) \neq U\} , \quad \forall e \in C$$

$$F^c(e) = \begin{cases} U \setminus F(e) & \text{if } e \in A \\ U & \text{Otherwise} \end{cases}$$

Definition 1.17. Let (F, A) and (G, B) be two soft sets over U . The difference of (F, A) and (G, B) , denoted by $(F, A) \setminus (G, B)$, is defined as $(F \setminus G, C)$ where

$$C = A \setminus \{e \in A \cap B \mid F(e) \subseteq G(e)\} , \quad \forall e \in C$$

$$(F \setminus G)(e) = \begin{cases} F(e) \setminus G(e) & \text{if } e \in A \cap B \\ F(e) & \text{Otherwise} \end{cases}$$

Example 1.18. Recall that the soft set (F, A) describing the “attractiveness of the houses” and defined by $A = \{e_2, e_3, e_4, e_5, e_7\}$

$$F(e_2) = \{h_2, h_3, h_5\} , \quad F(e_3) = \{h_2, h_4\} , \quad F(e_4) = \{h_1\} , \quad F(e_5) = U ,$$

$$F(e_7) = \{h_3, h_5\}$$

In addition, we assume that the “attractiveness of the houses” is described by the soft set (G, B) , where

$$G(e_1) = \{h_3, h_5\} , \quad G(e_2) = \{h_4\} , \quad G(e_3) = \{h_2, h_4\} , \quad G(e_5) = \{h_2, h_3, h_4, h_5\} ,$$

$$G(e_6) = G(e_7) = \{h_3\}$$

Then by a direct computation, we can readily obtain $(F, A) \cap (G, B)$, $(F, A) \cup (G, B)$, $(F, A)^c$ and $(F, A) \setminus (G, B)$ as follows.

(i) $(F, A) \cap (G, B) = (F \cap G, \{e_3, e_4, e_5, e_7\})$ where

$$(F \cap G)(e_3) = \{h_2, h_4\} , \quad (F \cap G)(e_4) = \{h_1\} , \quad (F \cap G)(e_5) = \{h_2, h_3, h_4, h_5\} , \\ (F \cap G)(e_7) = \{h_3\}$$

(ii) $(F, A) \cup (G, B) = (F \cup G, \{e_1, e_2, e_3, \dots, e_7\})$ where

$$(F \cup G)(e_1) = \{h_3, h_5\} , \quad (F \cup G)(e_2) = \{h_2, h_3, h_4, h_5\} , \quad (F \cup G)(e_3) = \{h_2, h_4\} , \\ (F \cup G)(e_4) = \{h_1\} , \quad (F \cup G)(e_5) = U , \quad (F \cup G)(e_6) = \{h_3\} , \quad (F \cup G)(e_7) = \{h_3, h_5\}$$

(iii) $(F, A)^c = (F^c, \{e_1, e_2, \dots, e_8\})$ where

$$F^c(e_1) = U , \quad F^c(e_2) = \{h_1, h_4\} , \quad F^c(e_3) = \{h_1, h_3, h_5\} , \quad F^c(e_4) = \{h_2, h_3, h_4, h_5\} , \\ F^c(e_6) = U , \quad F^c(e_7) = \{h_1, h_4\} \text{ and } F^c(e_8) = U$$

(iv) $(F, A) \setminus (G, B) = (F \setminus G, \{e_2, e_5, e_7\})$ where

$$(F \setminus G)(e_2) = \{h_2, h_3, h_5\} , \quad (F \setminus G)(e_5) = \{h_1\} \text{ and } (F \setminus G)(e_7) = \{h_5\}$$

Proposition 1.19. *Let (F, A) and (G, B) be two soft sets over U . Then*

$(F, A) = (F, A)^C$ if and only if $(F, A)^c \cap (F, A) = (\phi, \phi)$ and $(F, A)^c \cup (F, A) = (U, E)$.

Proof: *If $(G, B) = (F, A)^c$, then we see by definition that $(F, A)^c \cap (F, A) = (\phi, \phi)$ and $(F, A)^c \cup (F, A) = (U, E) = (F, A) \cup (F^c, \{e \in A \mid F(e) = U\}^c) = (U, E)$. Whence, the necessity is true.*

Conversely, *assume that $(F, A)^c \cap (F, A) = (\phi, \phi)$ and $(F, A)^c \cup (F, A) = (U, E)$.*

The latter means that $E = A \cup B$. Moreover, we obtain that $F(e) = U$ for all $e \in A \setminus B$ and $G(e) = U$ for all $e \in B \setminus A$.

For any $e \in A \cap B$, it follows from $(F, A)^c \cap (F, A) = (\phi, \phi)$ and $(F, A)^c \cup (F, A) = (U, E)$ that $F(e) \cup G(e) = U$ and $F(e) \cap G(e) = \phi$. As neither $F(e)$ nor $G(e)$ is empty, this forces that $B = \{e \in A \mid F(e) = U\}^c$.

For any $e \in B$, if $e \in A$, then $G(e) = F(e)^c = F^c(e)$.

if $e \in B \setminus A$, then $G(e) = U = F^c(e)$.

This implies that $(F, A)^c = (F^c, \{e \in A \mid F(e) = U\}^c) = (G, B)$

Corollary 1.20. *For any soft set (F, A) over U , we have that*

$$((F, A)^c)^c = (F, A)$$

Proof: *Note that $(F, A)^c \cap (F, A) = (\phi, \phi)$ and $(F, A)^c \cup (F, A) = (U, E)$. It therefore follows from Proposition 1.19 that $(F, A) = ((F, A)^c)^c$ as desired.*

With the above corollary, we can prove the De Morgan's laws of soft set

1.1.1 DeMorgan's laws

In this sub section, we show that the following De Morgan's type of results hold in soft set theory for different types of union , intersection , complements operation.

Theorem 1.21. (DeMorgan's laws). *For any soft sets (F, A) and (G, B) over U . we have that*

$$(1) [(F, A) \cup (G, B)]^r = (F, A)^r \cap (G, B)^r$$

$$(2) [(F, A) \cap (G, B)]^r = (F, A)^r \cup (G, B)^r$$

Proof: (1) Let $(F, A) \cup (G, B) = (H, C)$ where $H(c) = F(c) \cap G(c)$ for all $c \in C = A \cap B \neq \phi$. since $((F, A) \cup (G, B))^r = (H, C)^r$ by definition $H^r(c) = U - [F(c) \cup G(c)] = [U - F(c)] \cup [U - G(c)]$ for all $c \in C$.

Now $(F, A)^r \cap (G, B)^r = (F^r, A) \cap (G^r, B) = (K, C)$ where $C = A \cap B$. So by definition, we have

$$\begin{aligned} K(c) &= F^r(c) \cap G^r(c) = (U - F(c)) \cap (U - G(c)) \\ &= H^r(c) \text{ for all } c \in C. \end{aligned}$$

$$\text{Hence } [(F, A) \cap (G, B)]^r = (F, A)^r \cup (G, B)^r$$

(2) Let $[(F, A) \cap (G, B)]^r = (H, C)$ where $H(c) = F(c) \cap G(c)$ for all $c \in C = A \cap B \neq \phi$. since $((F, A) \cap (G, B))^r = (H, C)^r$ by definition

$$H^r(c) = U - (F(c) \cap G(c)) = [U - F(c)] \cup [U - G(c)] \text{ for all } c \in C.$$

Now $(F, A)^r \cup (G, B)^r = (F^r, A) \cup (G^r, B) = (K, C)$ where $C = A \cap B$. So by definition, we have

$$\begin{aligned} K(c) &= F^r(c) \cup G^r(c) = (U - F(c)) \cup (U - G(c)) \\ &= H^r(c) \text{ for all } c \in C. \end{aligned}$$

$$\text{Hence } [(F, A) \cap (G, B)]^r = (F, A)^r \cup (G, B)^r$$

1.2 Soft Set Functions

Definition 1.22. Let (F, A) and (G, B) be two non-empty soft sets. Then soft set relation $f : (F, A) \rightarrow (G, B)$ is called a soft set function if every element in domain has a unique element in the range. If $F(a)fG(b)$ then we write $f(F(a)) = G(b)$.

Example 1.23. Let $U = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7\}$

$$A = \{a_1, a_2, a_3, a_4\}$$

$$B = \{b_1, b_2\}$$

Consider the soft sets (F, A) and (G, B) defined by

$F(a_1) = \{p_1, p_2, p_5\}$, $F(a_2) = \{p_2, p_3, p_4\}$, $F(a_3) = \{p_1, p_2\}$, $F(a_4) = \{p_5, p_6\}$, and $G(b_1) = \{p_1, p_2, \}$, $G(b_2) = \{p_2, p_5, p_6\}$. Then a soft set function $f : (F, A) \rightarrow (G, B)$ is given by

$$f = \{F(a_1) \times G(b_1), F(a_2) \times G(b_1), F(a_3) \times G(b_2), F(a_4) \times G(b_2)\}$$

Definition 1.24. A function f from (F, A) to (G, B) is called injective (one-one) if $F(a) \neq F(b)$ implies $f(F(a)) \neq f(F(B))$.

Definition 1.25. A function f from (F, A) to (G, B) is called surjective (onto) if $ran f = (G, B)$

Definition 1.26. A function which both injective and surjective is called a bijective function.

Chapter 2

Soft Groups

Let G be a group and A be a nonempty set. R will refer to an arbitrary binary relation between an element of A and an element of G . A set-valued function $F : A \rightarrow P(G)$ can be defined as $F(x) = \{y \in G \mid (x, y) \in R\}$ for all $x \in A$. The pair (F, A) is a soft set over G . Defining a set valued function from A to G also defines a binary relation R on $A \times G$, given by $R = \{(x, y) \in A \times G : y \in F(x)\}$. The triple (A, G, R) is referred to as an approximation set.

Definition 2.1. Let (F, A) be a soft set over G . Then (F, A) is called a soft group over G if and only if $F(x)$ is a subgroup of G for all $x \in A$.

Example 2.2. Suppose that $G = A = S_3 = \{e, (12), (13), (23), (123), (132)\}$ and that we defined the set valued function $F(x) = \{y \in G : xRy \Leftrightarrow y = x^n, n \in N\}$. Then the soft group (F, A) is a parameterized family $\{F(x) : x \in A\}$ of subsets, which gives us a collection of subgroups of G .

$$\begin{aligned} F(e) &= \{e\} \quad , \quad F(12) = \{e, (12)\} \quad , \quad F(13) = \{e, (13)\} \quad , \quad F(23) = \{e, (23)\} \quad , \\ F(123) &= F(132) = \{e, (123), (132)\} \end{aligned}$$

Definition 2.3. Let (F, A) be a soft set over G . Then, for all $x \in A$

- i) (F, A) is called an identity soft group over G if $F(x) = \{e\}$, where e is the identity element of G .
- ii) (F, A) is called an absolute soft group over G if $F(x) = G$.

Definition 2.4. For a soft set (F, A) the set $Supp(F, A) = \{x \in A \mid F(x) \neq \phi\}$ is called the Support of the soft set (F, A) . The null soft set is a soft set with an empty support, and a soft set (F, A) is non-null if $Supp(F, A) \neq \phi$.

Definition 2.5. Let (F, A) and (H, K) be two soft group over G . Then (H, K) is a soft subgroup of (F, A) , written $(H, K) < (F, A)$ if

- (i) $K \subset A$
- (ii) $H(x) < F(x)$ for all $x \in K$

Theorem 2.6. Let (F, A) and (G, A) be two soft groups over G .

- (1) If $F(x) \subseteq H(x)$ for all $x \in A$, then (F, A) is a soft subgroup of (H, A) .
- (2) If $E = \{e_G\}$, and (F, A) and (F, G) are both soft groups over G , then (F, A) is a soft subgroup of (F, G) .

Proposition 2.7. Let (F, A) and (H, B) be two soft groups over G , Then their intersection $(F, A) \cap (H, B)$ is also a soft group over G .

Proposition 2.8. *Let (F, A) and (H, B) be two soft groups over G . if $A \cap B = \phi$, Then $(F, A) \cup (H, B)$ is also a soft group over G .*

Proof: *Since $A \cap B = \phi$, it follows that either $x \in A - B$ or $x \in B - A$ for all $x \in C$. let $(F, A) \cup (H, B) = (U, C)$ If $x \in A - B$ then $U(x) = F(x) < G$ and $x \in B - A$ then $U(x) = H(x) < G$ Thus $(F, A) \cup (H, B)$ is also a soft group over G .*

Example 2.9. Let $G = \{0, a, b, c\}$ be the Klein-4 group with the operation table given below.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Let $A = G$ and the soft set (F, A) over G , where $F : A \rightarrow P(G)$ is a set-valued function defined by $F(x) = \{y \in G \mid x\alpha y \Leftrightarrow y = nx \text{ for some } n \in N\}$

for all $x \in A$. Here, $nx = x + x + \dots + x$ means the n-fold sum of x and $0x = 0$, Then $F(0) = \{0\}$, $F(a) = \{0, a\}$, $F(b) = \{0, b\}$, $F(c) = \{0, c\}$, which are all subgroups of G . Hence, (F, A) is a soft group over G .

Let $B = G$ and the soft set (H, B) over G , where $H : B \rightarrow P(G)$ is defined by $H(x) = \{0\} \cup \{y \in G \mid x\alpha y \Leftrightarrow x + y = b\}$

Then $H(0) = \{0, b\}$, $H(a) = \{0, c\}$, $H(b) = \{0\}$, $H(c) = \{0, a\}$ which are all subgroups of G . Hence, (H, B) is a soft group over G .

2.1 Normal soft groups

In this section, we define normal soft group and prove some of their related results.

Definition 2.10. A soft group (F, A) over G is called a normal soft group over G if $F(x)$ is a normal subgroup of G for all $x \in A$.

Example 2.11. Consider the group $D_4 = \{e, a, a^2, a^3, b, ab, a^2b, a^3b\}$ which is generated by two elements a and b satisfying the relations $o(a) = 4$, $o(b) = 2$ and $ba = a^3b$.

Consider the set of parameters $A = \{t_1, t_2, t_3\}$. A mapping $F : A \rightarrow P(D_4)$ such that $F(t_1) = \{e, a, a^2, a^3\}$, $F(t_2) = \{e, a^2, b, a^2b\}$ and $F(t_3) = \{e, ab, a^2, a^3b\}$

For each parameter, $F(t)$ is a normal subgroup of D_4 . This implies that the pair (F, A) is a normal soft group over G .

Corollary 2.12. Let (F, A) be a normal soft group over G . Then (F, A) is commutative with every soft set over G .

Definition 2.13. Let (F, A) be a soft group over G and (H, B) be a soft subgroup of (F, A) . Then, we say that (H, B) is a normal soft subgroup of (F, A) , written $(H, B) \triangleleft (F, A)$, if $H(x)$ is a normal subgroup of $F(x)$; i.e, $H(x) \triangleleft F(x)$ for all $x \in B$

2.2 Abelian soft groups and Abelian soft subgroups

In this section, we define Abelian soft groups and Abelian soft Subgroups of a soft group and check out their properties.

Definition 2.14. A soft group (F, A) over G is said to be abelian soft group over G if each $F(\alpha)$ is an abelian subgroup of G for all $\alpha \in A$.

Example 2.15. Consider the group $S_3 = \{e, x, y, y^2, xy, xy^2\}$ which is generated by two elements x and y satisfying the relation $o(x) = 2$, $o(y) = 3$, $xy = y^2x$ and $yx = xy^2$. Consider the set of parameters $A = \{e, \alpha, \beta, \gamma\}$. A mapping $F : A \rightarrow P(S_3)$ such that $F(\alpha) = \{e, x\}$, $F(\beta) = \{e, y, y^2\}$ and $F(\gamma) = \{e, xy\}$. Then (F, A) is a soft group over S_3 . since for each parameter $\alpha \in A$, $F(\alpha)$ is an abelian subgroup of S_3 . Hence (F, A) is an abelian soft group over S_3 . It is interesting to not that S_3 is not abelian.

Definition 2.16. Let (F, A) be a soft group over G and (H, B) be a soft subgroup of (F, A) . Then we say that (H, B) is an abelian soft subgroup of (F, A) if $H(x)$ is an abelian subgroup of $F(x)$ for all $x \in B$.

Theorem 2.17. Every soft subgroup (H, B) of an abelian soft group (F, A) over G is a normal soft subgroup of (F, A) .

Proof: Since each $F(\alpha)$ for all $\alpha \in A$ is an abelian subgroup of G and each $H(\alpha)$ is a subgroup of $F(\alpha)$ for all $\alpha \in B$. It is well known that a subgroup of an abelian group is a normal subgroup. This implies that $H(\alpha)$ is a normal subgroup $F(\alpha)$ for all $\alpha \in B$. Hence $(H, B) \triangleleft (F, A)$.

2.3 Cartesian product of soft groups

In this section, prove some results by using cartesian product of soft groups.

Definition 2.18. Let (F, A) and (H, B) be two soft sets over G and K , respectively, and let $f : G \rightarrow K$ and $g : A \rightarrow B$ be two functions. Then we say that (f, g) is a soft homomorphism, if the following conditions are satisfied:

1. f is a homomorphism from G to K .
2. g is mapping from A onto B .
3. $f(F(x)) = H(g(x))$

In this definition, if f is an isomorphism G to K and g is a one-to-one mapping from A on to B , then we say that (f, g) is a soft isomorphism and that (F, A) is soft isomorphic to (H, B) . The latter is denoted by $(F, A) \simeq (H, B)$

Definition 2.19. Let (F, A) and (H, B) be two soft groups over G and K , respectively. Then the cartesian product of soft groups (F, A) and (H, B) is denoted by $(F, A) \times (H, B) = (U, A \times B)$ and U is defined as $U(a, b) = F(a) \times H(b)$ for all $(a, b) \in A \times B$.

Corollary 2.20. Let (F, A) and (H, B) be two normal soft groups over G and K respectively. Then the cartesian product $(F, A) \times (H, B)$ is a normal soft group over $G \times K$ and $(F, A) \times (H, B)$ is soft isomorphic to $(H, B) \times (F, A)$.

Theorem 2.21. *Let (F, A) and (H, B) be two soft groups over G and K , respectively. Then the cartesian product $(F, A) \times (H, B)$ is a soft group over $G \times K$ and $(F, A) \times (H, B)$ is soft isomorphic to $(H, B) \times (F, A)$.*

Proof: Now we show that $(f, g) : (F, A) \times (H, B) \rightarrow (H, B) \times (F, A)$ is a soft isomorphism. That is $(f, g) : (U, A \times B) \rightarrow (W, B \times A)$ is a soft isomorphism where $W(b, a)$ is defined as $W(b, a) = H(b) \times F(a)$. Actually, we prove three conditions.

1. We show that $f : G \times K \rightarrow K \times G$ is an isomorphism. Let f be a function defined by $f(g, k) = (k, g)$. Then obviously f is an isomorphism.
2. Now we show that $g : A \times B \rightarrow B \times A$ is a bijective mapping. The mapping g is defined by $g(a, b) = (b, a)$ then obviously g is a bijective mapping.
- 3.

$$\begin{aligned}
 f(U(a, b)) &= f(F(a) \times H(b)) \\
 &= f(\{(g, k) : g \in F(a), k \in H(b)\}) \\
 &= \{(k, g) : k \in H(b), g \in F(a)\} \\
 &= H(b) \times F(a) \\
 &= W(b, a) \\
 &= W(g(a, b)) \text{ for all } (a, b) \in A \times B
 \end{aligned}$$

This implies that $(f, g) : (F, A) \times (H, B) \rightarrow (H, B) \times (F, A)$ is a soft isomorphism. Hence, $(F, A) \times (H, B) \simeq (H, B) \times (F, A)$.

2.4 Cyclic Soft Groups

In this section, we define cyclic soft groups, cyclic soft subgroups and check out their properties.

Definition 2.22. A soft group (F, A) over G is called a cyclic soft group over G if each $F(\alpha)$ is a cyclic subgroup of G for all $\alpha \in A$.

Example 2.23. Consider the group $D_4 = \{e, a, a^2, a^3, b, ab, a^2b, a^3b\}$ which is generated by two elements a and b satisfying the relations $o(a) = 4$, $o(b) = 2$ and $ba = a^3b$.

Consider the set of parameters $A = \{t_1, t_2, t_3, t_4, t_5, t_6\}$. A mapping $F : A \rightarrow P(D_4)$

such that $F(t_1) = \{e, a^2\}$, $F(t_2) = \{e, b\}$, $F(t_3) = \{e, ab\}$, $F(t_4) = \{e, a^2b\}$,

$F(t_5) = \{e, a^3b\}$, $F(t_6) = \{e, a, a^2, a^3\}$ then (F, A) is a soft group over D_4 .

since for each parameter $\alpha \in A$, $F(\alpha)$ is a cyclic subgroup of D_4 .

Hence (F, A) is a cyclic soft group over D_4 .

Definition 2.24. Let (F, A) be a soft group over G . Then a soft subgroup (H, B) of (F, A) is called a cyclic soft subgroup of (F, A) if $H(\alpha)$ is a cyclic subgroup of $F(\alpha)$ for all $\alpha \in A$.

Theorem 2.25. Every cyclic soft group (F, A) over G is an abelian soft group over G .

Proof: Let (F, A) be a cyclic soft group over G . Then each $F(\alpha)$ is a cyclic subgroup of G for all $\alpha \in A$, but then each cyclic subgroup is abelian subgroup of G . Hence each $F(\alpha)$ is an abelian subgroup of G for all $\alpha \in A$. This implies that (F, A) is an abelian soft group over G .

Theorem 2.26. *Let (F, A) be a soft group over G . If a cyclic soft subgroup (H, B) of (F, A) is a normal soft subgroup of (F, A) , then every soft subgroup of (H, B) is a normal soft subgroup of (F, A) .*

Proof: Suppose cyclic soft subgroup (H, B) of (F, A) is a normal soft subgroup of (F, A) . Let (K, C) be a soft subgroup of (H, B) , this implies that (K, C) is a cyclic subgroup of (H, B) and hence a cyclic subgroup of (F, A) . We show that (K, B) is a normal soft subgroup of (F, A) . We have $H(\alpha)$ is a cyclic subgroup of $F(\alpha)$ for all $\alpha \in B$. But, $H(\alpha)$ is a normal subgroup of $F(\alpha)$ for all $\alpha \in B$. Hence, (K, C) , a soft subgroup of (H, B) is a normal soft subgroup of (F, A) .

2.5 Soft coset of a soft group

In this section, we define the soft coset of a soft subgroup of a soft group, order of a soft group, soft index, partition of a soft group and prove their related results.

Definition 2.27. Let (F, A) be a soft group over G and (H, B) is any soft subgroup of (F, A) . Let $a \in \cap F(x)$. Then the soft set (Ha, B) defined as $(Ha(b) = (H(b))$ a for all $b \in B$ is called a soft right coset of (H, B) in (F, A) generated by a . Similarly, the soft set (aH, B) is called a soft left coset of (H, B) in (F, A) generated by a .

Example 2.28. Consider the group $D_8 = \{e, a, a^2, a^3, b, ab, a^2b, a^3b\}$ which is generated by two elements a and b satisfying the relations $o(a) = 4$, $a^2 = b^2$ and $ba = a^3b$.

Consider the set of parameters $A = \{t_1, t_2, t_3, t_4\}$. A mapping $F : A \rightarrow P(D_8)$ such that $F(t_1) = \{e, a^2\}$, $F(t_2) = \{e, a, a^2, a^3\}$, $F(t_3) = \{e, ab, a^2, a^3b\}$ and $F(t_4) = \{e, b, a^2, a^2b\}$. For each parameter, $F(t)$ is subgroup of D_8 . Hence (F, A) is a soft group over D_8 .

since $\cap F(t_i) = \{e, a^2\}$ for all $t_i \in A$.

Let $B = \{t_2, t_3, t_4\}$ and consider (H, B) be any soft subgroup of (F, A) such that $H(t_2) = \{e, a^2\}$, $H(t_3) = \{e, ab, a^2, a^3b\}$, $H(t_4) = \{e, b, a^2, a^2b\}$. Now we find soft left coset as follows: For identity element we have,

$$({}_eH, B) = (H, B)$$

and

$$({}_{a^2}H, B) = \{a^2(H(t_2)) : \forall t \in B\}$$

Now we find $a^2H(t)$ for all $t \in B$ as follows

$a^2(H(t_2)) = H(t_2)$, $a^2(H(t_3)) = H(t_3)$, $a^2(H(t_4)) = H(t_4)$. This implies that a $(H, B) = (H, B)$. Hence, the only soft left coset of (H, B) in (F, A) is itself (H, B) .

Similarly, we can find soft right coset which is (H, B) itself.

Definition 2.29. Let (F, A) be a soft group over G . Then the order of a soft group (F, A) is the number of distinct subgroups of G

Theorem 2.30. Let (F, A) be a soft group over a finite group G and $F(x) \in (F, A)$. Then one has the following

1. The order of $F(x)$ divides the order of (F, A) . In particular, $F(x)^{|(F, A)|} = \{e\}$.
2. The order of (F, A) divides the order of G .

When G is a finite group, then the order (F, A) is a finite and the orders of elements of (F, A) divide the order of (F, A) . However when G is an infinite group, then the order of (F, A) can be finite or infinite.

Example 2.31. let (F, A) be soft set over S_3 , where $A = S_3$, and find the order of the elements of S_3

$$F(e)^1 = \{e\} \quad |F(e)| = 1 \quad F(12)^1 = \{(12)\} \quad F(12)^2 = \{e\} \quad |F(12)| = 2$$

$$F(13)^1 = \{(13)\} \quad F(13)^2 = \{e\} \quad |F(13)| = 2$$

$$F(23)^1 = \{(23)\} \quad F(23)^2 = \{e\} \quad |F(23)| = 2$$

$$F(123)^1 = \{(123)\} \quad F(123)^2 = \{(132)\} \quad F(123)^3 = \{e\} \quad |F(123)| = 3$$

$$F(132)^1 = \{(132)\} \quad F(132)^2 = \{(123)\} \quad F(132)^3 = \{e\} \quad |F(132)| = 3$$

Example 2.32. Let (F, N) be a soft group over group of integer numbers Z where F is a mapping from natural numbers N to $P(Z)$ such that $F(n) = nZ$ for all $n \in N$. There is no any positive integer m such that $F(n)^m = \{e\}$, so $F(n)$ has infinite order for all $n \in N - \{0\}$.

Theorem 2.33. Let (F, A) be a soft set over finite group G and $F(x) \in (F, A)$ for $x \in A$, Then the order of $F(x)$ is the least common multiple (LCM) of order of elements of $F(x)$

Proof: Let n be the order of $F(x)$. Then $F(x)^n = \{e\}$. This means that $a^n = e$ for all $a \in F(x)$. We know from classical group theory that $|a| \mid n$, namely, a , divides n for all $a \in F(x)$. Thus, n is common multiple of elements of $F(x)$. Let m be another common multiple of elements of $F(x)$, Then by reason of $a^m = e$ for all $a \in F(x)$, $F(x)^m = \{e\}$. However, since n is the least number that satisfies the condition $F(x)^n = \{e\}$ hence $n \mid m$.

Theorem 2.34. Let G be finite group (F, A) a soft group over G , and $F(x)$ and $F(y)$ the elements of (F, A) , Then for all $x, y \in A$, one has the following:

1. $|F(x) \cap F(y)| \leq GCD(|F(x)|, |F(y)|)$ for $x, y \in A$
2. $|F(x) \cup F(y)| = LCM(|F(x)|, |F(y)|)$ for $x, y \in A$
3. $|F(x) \times F(y)| = |F(x)||F(y)|$ for $x, y \in A$.

Proof: We consider the following.

1. $F(x) \cap F(y)$ is subgroup of $F(x)$ and $F(y)$ so $|F(x) \cap F(y)| \mid |F(x)|$ and $|F(x) \cap F(y)| \mid |F(y)|$. It follows $|F(x) \cap F(y)| \leq GCD(|F(x)|, |F(y)|)$

Definition 2.35. The number of distinct soft left (or right) cosets of a soft subgroup (H, B) of a soft group (F, A) over G is called the soft index of (H, B) in (F, A) and is denoted by $[(F, A) : (H, B)]$.

Theorem 2.36. A soft subgroup (H, B) of a soft group (F, A) over G is a normal soft subgroup of (F, A) if and only if each soft left coset of (H, B) in (F, A) is a soft right coset of (H, B) in (F, A) .

Proof: Let (H, B) be a normal soft subgroup of (F, A) . Then obviously $({}_xH, B) = (H_x, B)$ for all $x \in \cap F(a)$. Thus, each soft left coset of (H, B) in (F, A) is a soft right coset of (H, B) in (F, A) .

Conversely, suppose that each soft left coset of (H, B) is a soft right coset. Then we have $({}_xH, B) = (H_x, B)$ for some $x \in \cap F(a)$. This implies that $x(H(b)) = (H(b))y$ for all $b \in B$. Since $e \in H(b)$ for all $b \in B$, so $xe = x \in x(H(b))$. Therefore, we have $x \in (H(b))x$ for all $b \in B$. This implies that $x \in (H(b))x$ and hence we have $x(H(b)) = (H(b))x$ for all $b \in B$. This shows that $({}_xH, B) = (H_x, B)$. Hence, (H, B) is a normal and obviously, normal soft subgroup of (F, A) .

Corollary 2.37. Let (F, A) be a soft group over G and (H, B) be a soft subgroup of (F, A) . Then for all $x \in \cap F(a)$, $|({}_xH, B)| = |(H_x, B)| = |(H, B)|$.

Definition 2.38. Let (F, A) be a soft set over U . Then by partition of (F, A) we mean a collection of all such soft subsets of (F, A) , that is $\{(F_i, A_i) : i \in I\}$ such that $(F, A) = \cup(F_i, A_i)$ and $(F_i, A_i) \cap (F_j, A_j) = \Phi$ for $i \neq j$ and Φ is a null soft set.

Theorem 2.39. *Let (F, A) be a soft group over G and (H, B) be a soft subgroup of (F, A) . Then for all $x, y \in \cap F(a)$ either $({}_xH, B) = ({}_yH, B)$ or $({}_xH, B) \cap ({}_yH, B) = \Phi$.*

Theorem 2.40. *Let (F, A) be a soft group over G and (H, A) be a soft subgroup of (F, A) . Then $\{({}_xH, B) : x \in \cap F(a)\}$ forms a partition of (F, A) over G .*

Theorem 2.41. *Let (H, B) be a soft subgroup of a soft group (F, A) over a finite group G . Then the order of (H, B) divides the order of (F, A) over G . In particular*

$$|(F, A)|[(F, A) : (H, B)]|(H, B)|$$

Proof: *Since G is finite so collection of subgroups of G is finite. Hence, the order of (F, A) is finite and the number of soft left cosets of (H, B) in (F, A) is finite.*

Let $\{({}_{x_1}H, B), ({}_{x_2}H, B), ({}_{x_3}H, B), \dots, ({}_{x_r}H, B)\}$ be the set of all distinct left cosets of (H, B) , where $x_i \in \cap F(x)$ $i = 1, 2, \dots, r$. Then by theorem 2.36, we have
 $(F, A) = \cup ({}_iH, B)$ *and* $({}_iH, B) \cap ({}_jH, B) = \Phi$ *for all* $i \neq j$, $1 \leq i, j \leq r$. *Hence,*
 $[(F, A) : (H, B)] = r$ *and*

$$|(F, A)| = |({}_{x_1}H, B)| + |({}_{x_2}H, B)| + |({}_{x_3}H, B)| + \dots + |({}_{x_r}H, B)|$$

By the Corollary 2.33, we have $|(H, A)| = |({}_{x_i}H, B)|$ for all $1 \leq i \leq r$.

Thus $|(F, A)| = |({}_{x_1}H, B)| + |({}_{x_2}H, B)| + |({}_{x_3}H, B)| + \dots + |({}_{x_r}H, B)|$, r times, that is $|(F, A)| = r|(H, B)|$. this implies that $|(F, A)| = [(F, A) : (H, B)]|(H, B)|$. Thus, the order of (H, B) divides the order of (F, A) .

2.6 Factor of a soft group

In this section, we introduce soft maximal normal subgroups, simple soft groups and factor soft groups. We establish some results as well.

Definition 2.42. Let (F, A) be a soft group over G . Then a normal soft subgroup (H, B) of (F, A) is said to be a soft maximal normal subgroup of (F, A) if there exists no proper normal soft subgroup (K, B) of (F, A) , which properly contains (H, B) . Thus, a normal soft subgroup (H, B) of (F, A) is soft maximal if and only if there exists no normal soft subgroup (K, B) of (F, A) such that $(H, B) \subset (K, B) \subset (F, A)$.

Example 2.43. Let $G = \{e, a, a^2, a^3, b, ab, a^2b, a^3b\}$ be a dihedral group of degree 4 generated by two elements a and b satisfying the relations $o(a) = 4$, $o(b) = 2$ and $ba = a^3b$. Consider the set of parameters $A = \{t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8\}$.

A mapping $F : A \rightarrow P(G)$ such that $F(t_1) = \{e\}$, $F(t_2) = \{e, a^2\}$
 $F(t_3) = \{e, b\}$, $F(t_4) = \{e, a^2b\}$, $F(t_5) = \{e, a, a^2, a^3, b, ab, a^2b, a^3b\}$
 $F(t_6) = \{e, a, a^2, a^3\}$, $F(t_7) = \{e, a^2, b, a^2b\}$, $F(t_8) = \{e, ab, a^2, a^3b\}$

For each parameter, $t \in A$, we have subgroup of G . This implies that (F, A) is a soft group over G . Let (H, B) be an other soft group over G such that $B = \{t_5, t_6, t_7, t_8\}$ and $H(t_5) = \{e, a, a^2, a^3, b, ab, a^2b, a^3b\}$, $H(t_6) = \{e, a, a^2, a^3\}$
 $H(t_7) = \{e, a^2, b, a^2b\}$, $H(t_8) = \{e, ab, a^2, a^3b\}$

This implies that (H, B) is a normal soft subgroup of (F, A) . This is a soft maximal normal subgroup of (F, A) .

Definition 2.44. Let (F, A) be a soft group over G . Then (F, A) is called a simple soft group over G if for any normal soft subgroup (H, B) of (F, A) we have either $(H, B) = (I, B)$ or $(H, B) = (F, A)$.

Example 2.45. Consider the group $S_3 = \{e, x, y, y^2, xy, xy^2\}$ which is generated by two elements x and y satisfying the relations $o(x) = 2$, $o(y) = 3$, $o(xy) = 2$, $xy = y^2x$ and $yx = xy^2$. Consider the set of parameters $A = \{\alpha, \beta, \gamma\}$. A mapping $F : A \rightarrow P(S_3)$ such that $F(\alpha) = \{e, x\}$, $F(\beta) = \{e, y, y^2\}$, $F(\gamma) = \{e, xy\}$. Then (F, A) is a soft group over S_3 . This is a simple soft group over S_3 .

Definition 2.46. Let (F, A) be a soft group over G and (H, B) be a normal soft subgroup of (F, A) . Then quotient or factor of (F, A) by (H, B) is denoted by $(F, A)/(H, B) = (L, B)$ and L is defined as $L(b) = F(b)/H(b)$ for all $b \in B$. Since each $F(b)/H(b)$ is a group under coset multiplication and is written as $L(b) = \{x(H(b)) : x \in F(b) \forall b \in B\}$. Then (L, B) is called a factor soft group of (F, A) by (H, B) .

Theorem 2.47. Let (F, A) be a soft group over G . A normal soft subgroup (H, B) of (F, A) is soft maximal if and only if the factor soft group $(F, A)/(H, B)$ is soft simple group.

Proof: Suppose (H, B) is soft maximal and $(F, A)/(H, B)$ is not soft simple group that is $(F, A)/(H, B)$ possesses proper normal soft subgroups. Let $(K, B)/(H, B)$ be a proper normal soft subgroup of $(F, A)/(H, B)$. Then (K, B) will be a normal soft subgroup of (F, A) containing (H, B) . Since $(K, B)/(H, B)$ is a proper soft subgroup of $(F, A)/(H, B)$, therefore $(H, B) \subset (K, B) \subset (F, A)$. Thus (K, B) is a normal soft subgroup of (F, A) and $(H, B) \subset (K, B) \subset (F, A)$. Therefore (H, B) is not soft maximal. This contradicts the hypothesis that (H, B) is maximal in (F, A) . Hence $(F, A)/(H, B)$ must be soft simple.

Conversely , let $(F, A)/(H, B)$ be a soft simple and let (H, B) be not soft maximal. Since (H, B) is not soft maximal, therefore there exists a normal soft subgroup (K, B) of (F, A) such that $(H, B) \subset (K, B) \subset (F, A)$. Then $(K, B)/(H, B)$ is a normal soft subgroup of $(F, A)/(H, B)$. This contradicts the hypothesis, hence (H, B) must be soft maximum.

2.7 Actions of Soft Group

In this section, we will describe the concept of soft action and exemplify it. In addition, we will establish some characterizations about it.

Example 2.48. Let G be a soft group and $H \subset G$ soft subgroup. Suppose that

$$X = \{Hg : g \in F(a)\}$$

is a right soft coset of H . Then for all $g_1 \in G$, the each mapping

$$\Pi_a : F(a)' \times F(a) \rightarrow F'(a)$$

is a group action, namely,

$$i) \quad \Pi_a(Hg, e) = H(ge) = Hg$$

$$\begin{aligned} ii) \quad \Pi_a(\Pi_a(Hg, g_1), g_2) &= \Pi_a(H(gg_1), g_2) = H((gg_1)g_2) = H(g(g_1g_2)) \\ &= Hg(g_1g_2) = \Pi_a(Hg, g_1g_2) \end{aligned}$$

for all $g_1, g_2 \in F(a)$.

Thus, the soft group G acts on the set of right soft cosets of H on the right.

Similarly, we can show that the soft group G acts on the set of left soft cosets of H on the left

Proposition 2.49. Denote $Orb_G(x)$ the orbit of $\sim$ containing x . clearly

$$Orb_G(x) = \{\Pi_a(g, x) : g \in F(a)\}$$

is a G -soft set.

Example 2.50. Every soft group G acts on itself as follows. Take $X = G$. Then for all $a \in A$, the mapping

$$\begin{aligned}\prod_a : F(a) \times F(a) &\rightarrow F(a) \\ (g, h) &\rightarrow \prod_a(g, h) = h\end{aligned}$$

is group action. Thus, this action is called trivial soft action

Example 2.51. Every soft group G acts on itself by multiplication as follows. Let us take the soft group G as soft set X . Then for all $a \in A$, the mapping

$$\begin{aligned}\prod_a : F(a) \times F(a) &\rightarrow F(a) \\ (g, h) &\rightarrow \prod_a(g, h) = gh\end{aligned}$$

is group action.

Example 2.52. Every soft group G acts on itself by conjugation as follows. take $X = G$. Then for all $a \in A$, the mapping

$$\begin{aligned}\prod_a : F(a) \times F(a) &\rightarrow F(a) \\ (g, h) &\rightarrow \prod_a(g, h) = ghg^{-1}\end{aligned}$$

is group action.

Remark 2.53. Note that if soft group G is abelian, then the soft action of G on itself by conjugation is trivial.

Definition 2.54. Let $X = (X, F', A)$ be a G -soft set. Then for all $a \in A$ and $x \in F'(a)$, the stabilizer of x is the

$$Stab_G(x) = \{g \in F(a) : \prod_a(g, x) = x\}$$

Definition 2.55. Let $G = (G, F, A)$ acts on itself by conjugation soft action. for all $a \in A$ and $h \in F(a)$, the set

$$C_G(h) = \{g \in F(a) : \prod_a(g, h) = \prod_a(h, g)\}$$

of the orbit of h is called the Centralizer of h in $F(a)$.

Furthermore, the intersection of all the centralizers of elements of $F(a)$ is denoted as follows

$$Z(G) = \{g \in F(a) : \prod_a(g, h) = \prod_a(h, g), \forall h \in F(a)\}$$

Proposition 2.56. The Centralizer C_G of h is a soft group over G .

Proof: For all $h \in F(a)$, we can define

$$\begin{aligned} C_G : G &\rightarrow P(G) \\ h &\rightarrow C_G(h) \end{aligned}$$

it is clear that $C_G(h)$ is a subgroup of G . Thus $C_G(h)$ is a soft group over G .

Conclusion

soft set theory has opened up keen insights and has a rich potential for application in many different fields such as ontology , data analysis , forecasting , simulation , decision making, medical science , rule mining , algebraic systems , optimization , and textures classification.

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